## ES 455 2014: Quiz 4 Solution

## Note that this is Q6 on the 2009 Midterm

## Instructions

1. Separate into groups of no more than three persons.
2. Only one submission is needed for each group. Late submission will not be accepted.
3. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

| Name | ID |
| :--- | :--- |
| Prapun |  |
|  |  |
|  |  |

4. Do not panic.

Consider a system which has 2 channels. We would like to find the call blocking probability via the Markov chain method. Assume that the total call request rate is 30 calls per hour and the average call duration is 6 ming.
a. Draw the Markov chain via discrete time approximation. Assume that the duration of each time slot is 1 millisecond. Don't forget to indicate the transition probabilities on the arrows.

$$
\begin{aligned}
\frac{1}{\mu} & =6 \mathrm{~min} \\
& =\frac{1}{10} \mathrm{hr} .
\end{aligned}
$$



$$
\lambda \delta=30 \times \frac{1 \times 10^{-3}}{3600}=8.33 \times 10^{-6}
$$

b. Find the long-term call blocking probability from the Markov cha $n$.
$A=\frac{\lambda}{\mu}=30 \times \frac{1}{10}$
$=3$ Eslangs $\begin{aligned} P_{0} \lambda \bar{\partial} & =P_{1} \mu \text { ot } \\ P_{1} & =A P_{0}\end{aligned}$
$\downarrow<\frac{1}{p_{0}}$
$p_{0}+p_{1}+p_{2}=1 \rightarrow p_{0}=\frac{1}{1+A+\frac{A^{2}}{2}} \Rightarrow p_{2}^{\prime \prime}=\frac{A^{2} / 2}{1+A+\frac{A^{2}}{2}}=\frac{9}{17}=0.529$
c. Use Erlang B formula, find the call blocking probability.

$$
P_{b}=\frac{\frac{A^{m}}{m!} \sqrt{m}=\frac{3^{2} / 2}{\sum_{i=1}^{m} \frac{A^{i}}{i!}}=1+3+3^{2} / 2}{2+6+9}=\frac{9}{17}=0.529
$$

